NTNU

Department of energy and process engineering

Examination paper for TEP4200, Mechanical Design, Operation and Maintenance of Hydraulic Machinery

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Examination date: 22nd May 2025

Examination time (from-to): 09:00 – 13:00

Permitted examination support material: D

No written and handwritten examination support materials are permitted. A simple calculator is permitted.

Other information:

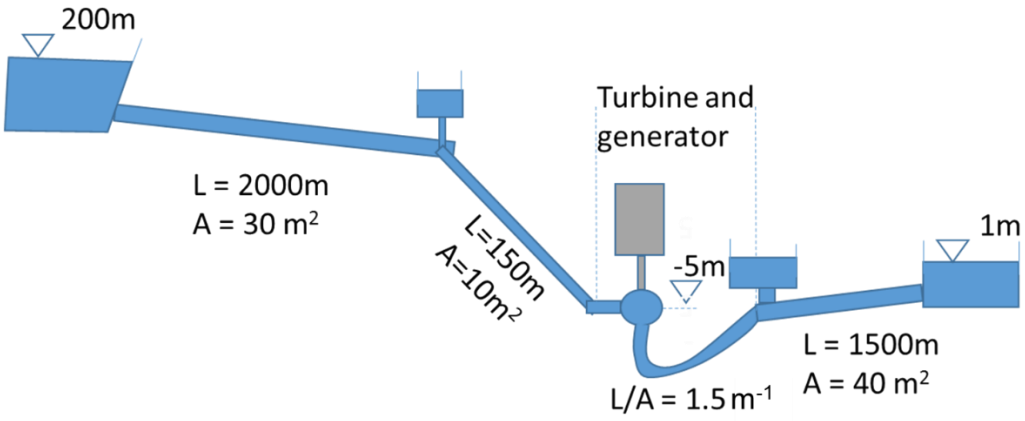
- You have to fill out and deliver all questions pages, 2 to 5, as your solution. No written notes should be delivered.
- For Multiple choice questions there can be more than one marked check box for each question. Correct answers are rewarded with one point (1p.) while incorrect answers give zero point (0p.).
- True or false are rewarded with one point (1p.) for correct answer, while incorrect answers give zero point (0p.).

Language: English

Number of pages (front page excluded):

Number of pages enclosed:

Problem 1. Turbine and System dynamics (max. 12 points)



Speed number	$\underline{\Omega} = \underline{\omega} \cdot \sqrt{\underline{Q}}$
Frequency and period of U-tube oscillation	$\omega = \sqrt{\frac{g}{A_{shaft} \cdot {}^{L}/_{A}}} \qquad T = \frac{2 \cdot \pi}{\omega}$
Penstock time constant and retardation/acceleration pressure	$T_w = \frac{Q}{g \cdot H} \cdot \Sigma\left({}^{L}/_{A}\right) \qquad \Delta H = \frac{1}{g} \cdot \sum \frac{L}{A} \cdot \frac{\Delta Q}{T_C}$
Up- or down-surge in the surge shaft	$\Delta z = \Delta Q \cdot \sqrt{\frac{\Sigma(L/A)}{g \cdot A_{shaft}}}$
Vapor pressure (absolute)	0,2 meter water column

1	A power plant is under construction with a lay-out as shown in the Figure above. The flow rate Q = 30 m^3/s, and the generator has 7 pole pairs.	Check box
1A.	Calculate the area of the upper surge shaft if Δz < 10 m. Neglect the head loss. (Single-choice question)	
i	The area of the upper surge shaft is: A_{Shaft} = 32 m^2	
ii	The area of the upper surge shaft is: A_{Shaft} = 43 m^2	
iii	The area of the upper surge shaft is: A_{Shaft} = 56 m^2	
iv	The area of the upper surge shaft is: A_{Shaft} = 61 m^2	X

$$\Delta z = \Delta Q \cdot \sqrt{\frac{\Sigma(L/A)}{g \cdot A_{shaft}}} \qquad A_{shaft} = \Delta Q^2 \cdot \frac{\Sigma(L/A)}{g \cdot \Delta z^2} = 30^2 \cdot \frac{2000/30}{9.81 \cdot 10^2} = 61\ m^2$$

1B.	Calculate the change of pressure in the draft tube at turbine level as the turbine closes in T_C = 11 seconds if there is no surge shaft installed. (Single-choice question)	
i	The change of pressure in the draft tube at turbine level is: ΔH = 5,9 m	
ii	The change of pressure in the draft tube at turbine level is: ΔH = 9,7 m	
iii	The change of pressure in the draft tube at turbine level is: ΔH = 10,8 m	X
iv	The change of pressure in the draft tube at turbine level is: ΔH = 11,5 m	

$$\Delta H = \frac{1}{g} \cdot \sum \frac{L}{A} \cdot \frac{\Delta Q}{T_C} = \frac{1}{g} \cdot \sum \left(\frac{1500}{40} + 1{,}5\right) \cdot \frac{30}{11} = 10{,}8\ m$$

1C. The surge shaft at the outlet of the draft tube has an area of 150 m^2. Calculate the down surge in surge shaft at shut down from a flow rate Q = 30 m^3/s (Single-choice question)		
i	The down surge in surge shaft is: Δz = 3,8 m	
ii	The down surge in surge shaft is: Δz = 4,2 m	
iii	The down surge in surge shaft is: Δz = 4,8 m	X
iv	The down surge in surge shaft is: Δz = 5,2 m	

$$\Delta z = \Delta Q \cdot \sqrt{\frac{\Sigma(L/A)}{g \cdot A_{shaft}}} = 30 \cdot \sqrt{\frac{\frac{1500}{40}}{9{,}81 \cdot 150}} = 4{,}8\ m$$

1D. The surge shaft at the outlet of the draft tube has an area of 150 m^2. Calculate the down surge period at shut down from a flow rate Q = 30 m^3/s (Single-choice question)		
i	The surge period is: T = 140 s	
ii	The surge period is: T = 150 s	X
iii	The surge period is: T = 160 s	
iv	The surge period is: T = 170 s	

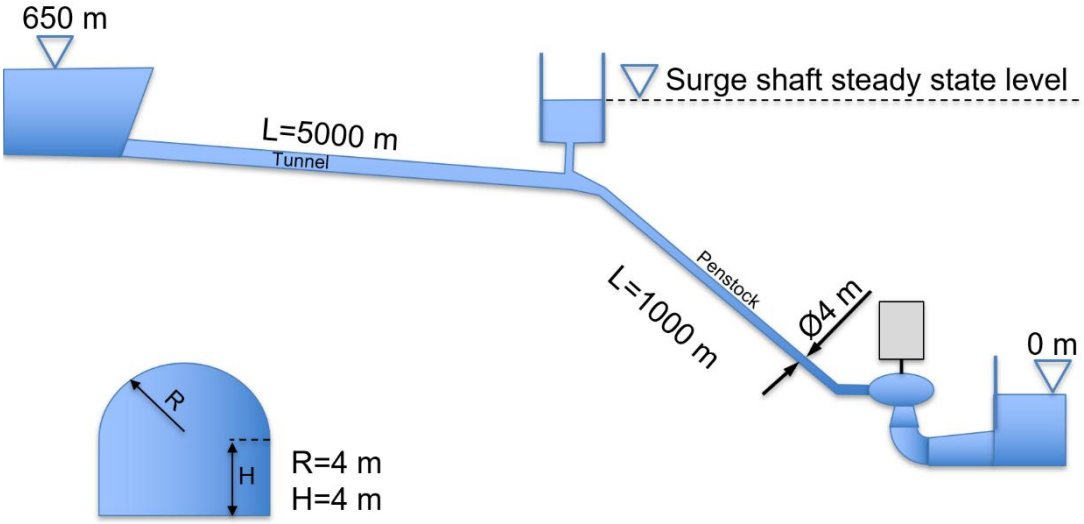
$$\omega = \sqrt{\frac{g}{A_{shaft} \cdot {}^{L}/_{A}}} = \sqrt{\frac{9{,}81 \cdot 40}{150 \cdot 1500}} = 0{,}042\ rad/s \qquad T = \frac{2 \cdot \pi}{\omega} = \frac{2 \cdot \pi}{0{,}042} = 150\ s$$

1E. Calculate the waterhammer upstream the turbine as the turbine shut down from the flow rate Q = 30 m^3/s in T_C = 8 seconds and there is no surge shaft installed. (Single-choice question)		
i	The waterhammer is: ΔH = 31 m	X
ii	The waterhammer is: ΔH = 36 m	
iii	The waterhammer is: ΔH = 39 m	
iv	The waterhammer is: ΔH = 41 m	

$$\Delta H = \frac{1}{g} \cdot \frac{\Delta Q}{T_C} \cdot \sum \left(\frac{L}{A}\right) = \frac{1}{g} \cdot \frac{30}{8} \cdot \sum \left(\frac{150}{10} + \frac{2000}{30}\right) = 31\ m$$

1F. True-False. The consequence of the change of pressure in the draft tube at turbine level as the turbine closes in T_C = 11 seconds if there no surge shaft will be: (True or False question)			
i	The change of pressure will not give any problems for the operation of the turbine	True	
ii	The change of pressure will result in cavitation at the outlet of the turbine runner		False
iii	The change of pressure will result in cavitation at the outlet of the draft tube		False
iv	The change of pressure will result in a water column separation in the draft tube		False

Problem 2. Head Loss (max. 10 points)



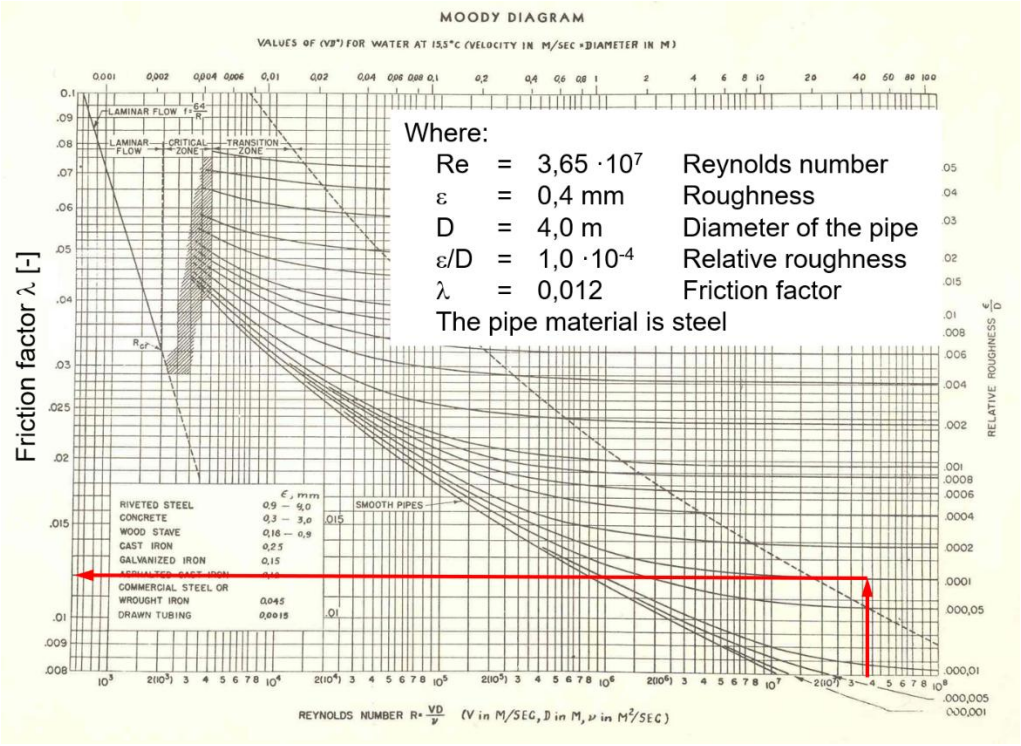
$$h_f = \frac{L}{M^2} \cdot \frac{c^2}{{R_h}^{4/3}} \qquad R_h = \frac{A}{P} \qquad h_f = \lambda \cdot \frac{L}{D} \cdot \frac{c^2}{2 \cdot g} \qquad Re = \frac{c \cdot D}{\upsilon}$$

Given data:

	Symbol		Unit
Gravity constant	g	9,81	m/s^2
Mannings constant	M	34	$m^{1/3}/s$
Kinematic viscosity	ν	$1{,}308 \cdot 10^{-6}$	m^2/s
Turbine efficiency	η_t	94	%
Generator efficiency	η_G	98	%
Transformer efficiency	η_T	98	%
Penstock material roughness	ε	0,4	mm
Diameter of the penstock	D	4,0	m
Flow rate	Q	150	m^3/s

2	Calculate the following for the hydropower plant with tunnel and penstock as shown in the figure above: (Multiple-choice question)	Check box
2a	Head loss in the tunnel; $H_f = 9{,}9$ m	
2b	Head loss in the tunnel; $H_f = 11{,}8$ m	X
2c	Head loss in the tunnel; $H_f = 14{,}1$ m	
2d	Head loss in the penstock; $H_f = 9{,}6$ m	
2e	Head loss in the penstock; $H_f = 15{,}1$ m	
2f	Head loss in the penstock; $H_f = 21{,}8$ m	X
2g	Turbine power output; $P_T = 814$ MW	
2h	Turbine power output; $P_T = 831$ MW	
2i	Turbine power output; $P_T = 853$ MW	X
2j	The steady state level in the surge shaft = 650 m	

2k	The steady state level in the surge shaft = 640,1 m	
2l	The steady state level in the surge shaft = 638,2 m	X
2m	The steady state level in the surge shaft = 635,9 m	
2n	The net head of the turbine; H_n = 616,4 m	X
2p	The net head of the turbine; H_n = 630,5 m	
2q	The net head of the turbine; H_n = 650,0 m	



Problem 3. Hydro Turbines (max. 10 points)

3	**The following statements are true or false.** (True or False statements)	True	False
3a	Hydropower is the largest source of renewable electrical energy in the world	X	
3b	Pump-turbines will increase the flexibility of the Norwegian energy system	X	
3c	At best efficiency operation of a Francis turbine, the rotational component of the absolute velocity, $c_{\theta 2}$ at the outlet to the runner is zero.	X	
3d	At Francis turbine part load operation, the rotational component of the absolute velocity, $c_{\theta 2}$ at the outlet of the runner is in the opposite direction of the peripheral velocity of the runner, u_2		X
3e	At best efficiency operation of a Pelton turbine, the rotational component of the absolute velocity at the inlet to the runner is zero.		X
3f	The relative friction losses in a laboratory scale turbine are lower than for the full scale turbine in a power plant		X
3g	The speed factor n_{ED} represent a relative relation to a turbine with a head: H=1 m and a diameter D_2=1 m		X
3h	Cavitation is defined as the transition from gas to fluid	X	
3i	Francis turbines are always submerged		X
3j	Pelton turbines are usually chosen for high specific speed numbers, Ω		X

Problem 4. Francis turbine performance diagram (max. 14 points)

Given data:

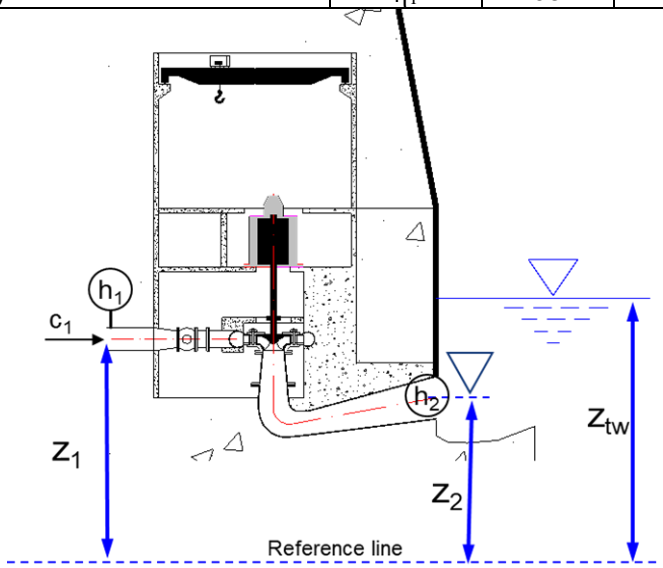
	Symbol		Unit
Inlet pressure on the model turbine	p_{in}	235	kPa
Outlet pressure on the model turbine	p_{out}	95	kPa
Atmospheric pressure	p_{atm}	93	kPa
Flow rate on the model turbine	Q_M	245	l/s
Gravity constant	g	9,8	m/s^2
Density of water	ρ	1000	kg/m^3
Velocity at the inlet of the turbine	C_{inlet}	3	m/s
Velocity at the outlet of the turbine	C_{outlet}	1	m/s
Level at the inlet	Z_{in}	1,5	m
Level at the outlet	Z_{out}	1	m
Model Turbine Torque	M_T	900	Nm
Model Turbine Speed	n_M	350	rpm
Prototype Turbine Speed	n_P	375	rpm
Generator efficiency	η_G	0,99	-
Model Turbine runner outlet diameter	D_M	0,35	m
Prototype Turbine runner outlet diameter	D_P	1,78	m
Number of runner blades	Z_{runner}	30	-
Number of guide vanes	$Z_{guide\ vanes}$	24	-

4	(Multiple-choice question)		Check box
4a	The net head of the model turbine is;	$H_n = 14,9$ m	
4b	The net head of the model turbine is;	$H_n = 15,2$ m	X
4c	The net head of the model turbine is;	$H_n = 15,7$ m	
4d	Model turbine efficiency;	$\eta_T = 89,3$ %	
4e	Model turbine efficiency;	$\eta_T = 90,4$ %	X
4f	Model turbine efficiency;	$\eta_T = 91,2$ %	
4g	Speed factor is;	$n_{ED} = 0,12$	
4h	Speed factor is;	$n_{ED} = 0,17$	X
4i	Speed factor is;	$n_{ED} = 0,19$	
4j	Flow factor is;	$Q_{ED} = 0,14$	
4k	Flow factor is;	$Q_{ED} = 0,16$	X
4l	Flow factor is;	$Q_{ED} = 0,18$	
4m	The flow rate of the prototype is:	$Q_P = 32\ m^3/s$	
4n	The flow rate of the prototype is:	$Q_P = 35\ m^3/s$	X
4o	The flow rate of the prototype is:	$Q_P = 38\ m^3/s$	
4p	The net head of the prototype is:	$H_n = 445$ m	
4q	The net head of the prototype is:	$H_n = 454$ m	X
4r	The net head of the prototype is:	$H_n = 473$ m	
4s	The rotor-stator interaction frequency in the model runner is: Hz	RSI = 99	
4t	The rotor-stator interaction frequency in the model runner is: Hz	RSI = 140	X
4u	The rotor-stator interaction frequency in the model runner is: Hz	RSI = 150	

Problem 5. Francis turbine net head (max. 6 points)

Given data:

	Symbol		Unit
Flow rate	Q	100	m^3/s
Gravity constant	g	9,81	m/s^2
Density of water	ρ	1000	kg/m^3
Diameter of the inlet pipe to the turbine	D_1	4	m
Cross section at the outlet of the draft tube	A_2	50	m^2
Level at the inlet to the turbine	z_1	100	m
Level at the outlet from the draft tube	z_2	95	m
Tail Water Level	z_{TWL}	115	m
Absolute Static pressure at the inlet to the turbine	h_1	450	m
Absolute Static pressure at the outlet from the draft tube	h_2	20	m
Atmospheric pressure	p_{atm}	100	kPa
Turbine efficiency	η_t	94	%
Generator efficiency	η_G	98	%
Transformer efficiency	η_T	98	%



5	(Multiple-choice question)		Check box
5a	The net head of the turbine is;	$H_n = 412$ m	
5b	The net head of the turbine is;	$H_n = 438$ m	X
5c	The net head of the turbine is;	$H_n = 425$ m	
5d	The net head of the turbine is;	$H_n = 452$ m	
5e	The generator output is;	$P_G = 387$ MW	
5f	The generator output is; ;	$P_G = 396$ MW	X
5g	The generator output is;	$P_G = 421$ MW	
5h	The generator output is;	$P_G = 430$ MW	
5i	The submergence of the turbine is;	$H_s = 12$ m	
5j	The submergence of the turbine is;	$H_s = 15$ m	X
5k	The submergence of the turbine is;	$H_s = 18$ m	
5l	The submergence of the turbine is;	$H_s = 21$ m	

Problem 6. Francis turbine - scale up from model to prototype (max. 8 points)

Given data:

	Symbol		Unit
Gravity constant	g	9,81	m/s^2
Density of water	ρ	1000	kg/m^3
Net head of the model turbine	H_{nM}	14,6	m
Net head of the prototype turbine	H_{nP}	377	m
Speed factor at best efficiency point	n_{ED}	0,17	
Flow factor at best efficiency point	Q_{ED}	0,16	
Model Turbine runner outlet diameter	D_M	0,35	m
Prototype Turbine runner outlet diameter	D_P	1,86	m
Number of runner blades	Z_{runner}	17	-
Number of guide vanes	$Z_{guide\ vanes}$	23	-

6	(Multiple-choice question)		Check box
6a	The speed of the prototype turbine is;	n_P = 300 rpm	
6b	The speed of the prototype turbine is;	n_P = 333 rpm	X
6c	The speed of the prototype turbine is;	n_P = 375 rpm	
6d	The flow rate of the prototype is:	Q_P = 28,1 m^3/s	
6e	The flow rate of the prototype is:	Q_P = 33,7 m^3/s	X
6f	The flow rate of the prototype is:	Q_P = 38,2 m^3/s	
6g	The Speed number is;	Ω = 0,22	
6h	The Speed number is;	Ω = 0,25	X
6i	The Speed number is;	Ω = 0,28	
6j	RSI measured in between the guide vanes and the runner is: RSI = 94 Hz		
6k	RSI measured in between the guide vanes and the runner is: RSI = 128 Hz		X
6l	RSI measured in between the guide vanes and the runner is: RSI = 153 Hz		

Problem 7 System dynamics

A power plant is under construction with a lay-out as shown in Figure 1. The selected turbine is a Francis turbine with design head of 400 m and design flow 40 m^3/s.

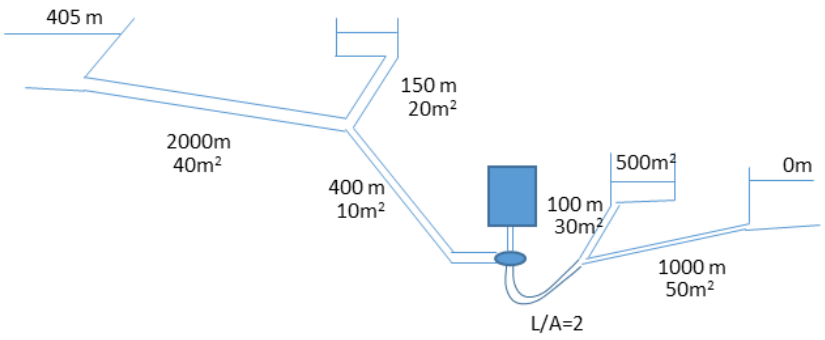


Figure 1 Water power plant

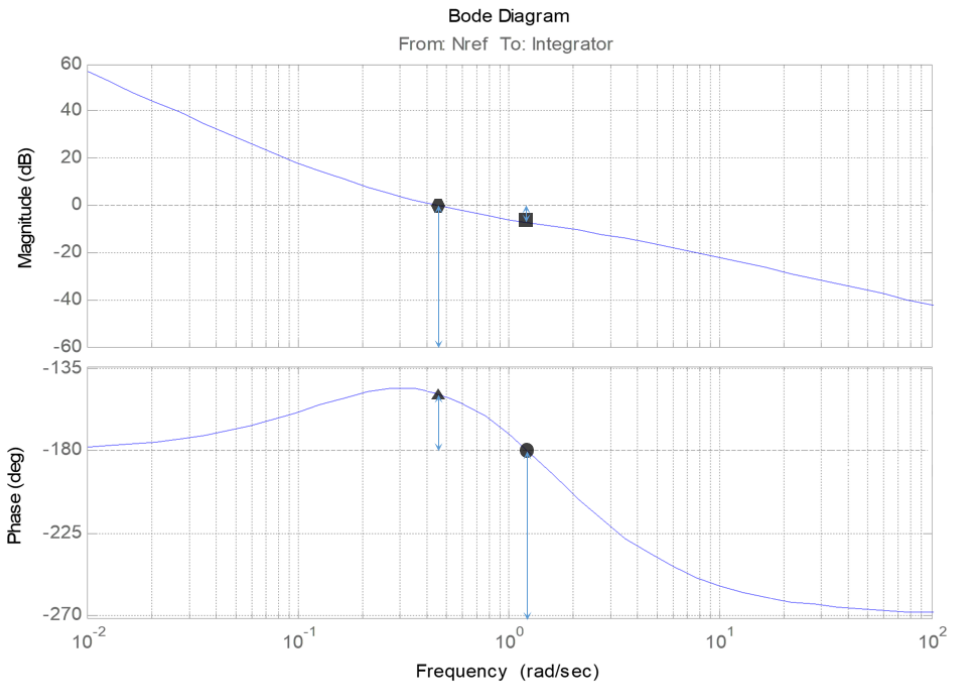
1	A power plant is under construction with a lay-out as shown in Figure 1. The selected turbine is a Francis turbine with design head of 400 m and design flow 40 m^3/s.	Check box
1A. The generator has 6 pole pairs. What is the synchronous speed of rotation? Grid frequency is 50 Hz.		
i	300 RPM	
ii	500 RPM	X
iii	600 RPM	
iv	750 RPM	
1B. The closing time of the wicket gate is T_C = 8 sec. Calculate the pressure rise at the turbine. Use ΔQ = 40 m^3/s.		
i	24.2 m	X
ii	49.7 m	
iii	29.3 m	
iv	27.7 m	
1C. Which of the following criteria should be fulfilled in order to neglect the elasticity?		
i	$T_c \gg \frac{2L}{a}$	X
ii	$T_c = T_a$	
iii	$T_c \ll \frac{2L}{a}$	
iv	$T_c = T_w$	
1D. The up-surge in the upstream surge shaft shall be restricted to a maximum level of Δz=10m. What will the area of the surge shaft be?		

i	93.8 m^2	X
ii	77.5 m^2	
iii	81.5 m^2	
iv	146.8 m^2	
1E. If T_a=5 sec, is the governor stability sufficient?		
i	Yes	X
ii	No	
iii	T_a has nothing to do with governor stability	
iv	Not enough information to determine governor stability	
1F. Time period of Tail water surge is		
i	216.7	X
ii	103.6	
iii	210.4	
iv	225.8	

Frequency of U-tube oscillation	$\omega = \sqrt{\frac{g}{A_{shaft}\, L/A}}$
Penstock time constant	$T_w = \frac{Q}{gH} \Sigma\, L/A$
Up-surge in the surge shaft	$\Delta z = \Delta Q \sqrt{\frac{\Sigma\, L/A}{g A_{shaft}}}$
Retardation pressure when Tc >>2L/a	$\Delta H = \frac{L}{g \cdot A} \cdot \frac{\Delta Q}{T_L}$

Problem 8 System Dynamics

A hydro power system has the following APF diagram or Bode diagram:



2	A hydro power system has the APF diagram or Bode diagram shown above:	
2a. What is the cross frequency?		
i	4.5 x 10^{-1} rad/s	X
ii	1.1 rad/s	
iii	1 x 10^{-2} rad/s	
iv	4.5 x 10^{0} rad/s	
2b. If the shapes on the curves in the diagram represent the height of the lines they are on, choose the shape that represents Phase Margin.		
i	■	
ii	▲	X
iii	●	
iv	⬢	
2c. If the shapes on the curves in the diagram represent the height of the lines they are on, choose the shape that represents Amplification Margin.		
i	■	X
ii	▲	
iii	●	
iv	⬢	
2d. Which of the following statements is True?		

i	Increasing the proportional constant ‘P’ on PID governor increases the cross frequency	X
ii	Increasing the proportional constant ‘P’ on PID governor decreases the cross frequency	
iii	The cross frequency is independent of proportional constant ‘P’.	
iv	None of the above	

Given equations:

Head loss: $h_f = \lambda \cdot \frac{L}{D} \cdot \frac{c^2}{2 \cdot g}$

Head loss: $h_f = \frac{L}{M^2} \cdot \frac{c^2}{{R_h}^{4/3}}$

Hydraulic radius: $R_h = \frac{A}{P}$

Reynolds number: $Re = \frac{c \cdot D}{\upsilon}$

Power output; $P = \rho \cdot g \cdot H_n \cdot Q \cdot \eta$

Frequency of U-tube oscillation: $\omega = \sqrt{\frac{g}{A_{shaft} \cdot {}^{L}/_{A}}}$

Period of U-tube oscillation: $T = \frac{2 \cdot \pi}{\omega}$

Penstock time constant: $T_w = \frac{Q}{g \cdot H} \cdot \Sigma\left({}^{L}/_{A}\right)$

Penstock retardation/acceleration pressure for Tc >>2L/a $\Delta H = \frac{1}{g} \cdot \sum \frac{L}{A} \cdot \frac{\Delta Q}{T_C}$

Up- or down-surge in the surge shaft: $\Delta z = \Delta Q \cdot \sqrt{\frac{\Sigma(L/A)}{g \cdot A_{shaft}}}$

Reduced numbers; $\underline{c} = \frac{c}{\sqrt{2 \cdot g \cdot H}}$ $\underline{u} = \frac{u}{\sqrt{2 \cdot g \cdot H}}$ $\underline{w} = \frac{w}{\sqrt{2 \cdot g \cdot H}}$ $\underline{Q} = \frac{Q}{\sqrt{2 \cdot g \cdot H}}$ $\underline{\omega} = \frac{\omega}{\sqrt{2 \cdot g \cdot H}}$

$\underline{\Omega} = \underline{\omega} \cdot \sqrt{\underline{Q}}$ $n_{ED} = \frac{n \cdot D}{\sqrt{E}}$ $n_q = n \cdot \frac{\sqrt{Q}}{H^{0,75}}$ $Q_{ED} = \frac{Q}{D^2 \cdot \sqrt{E}}$

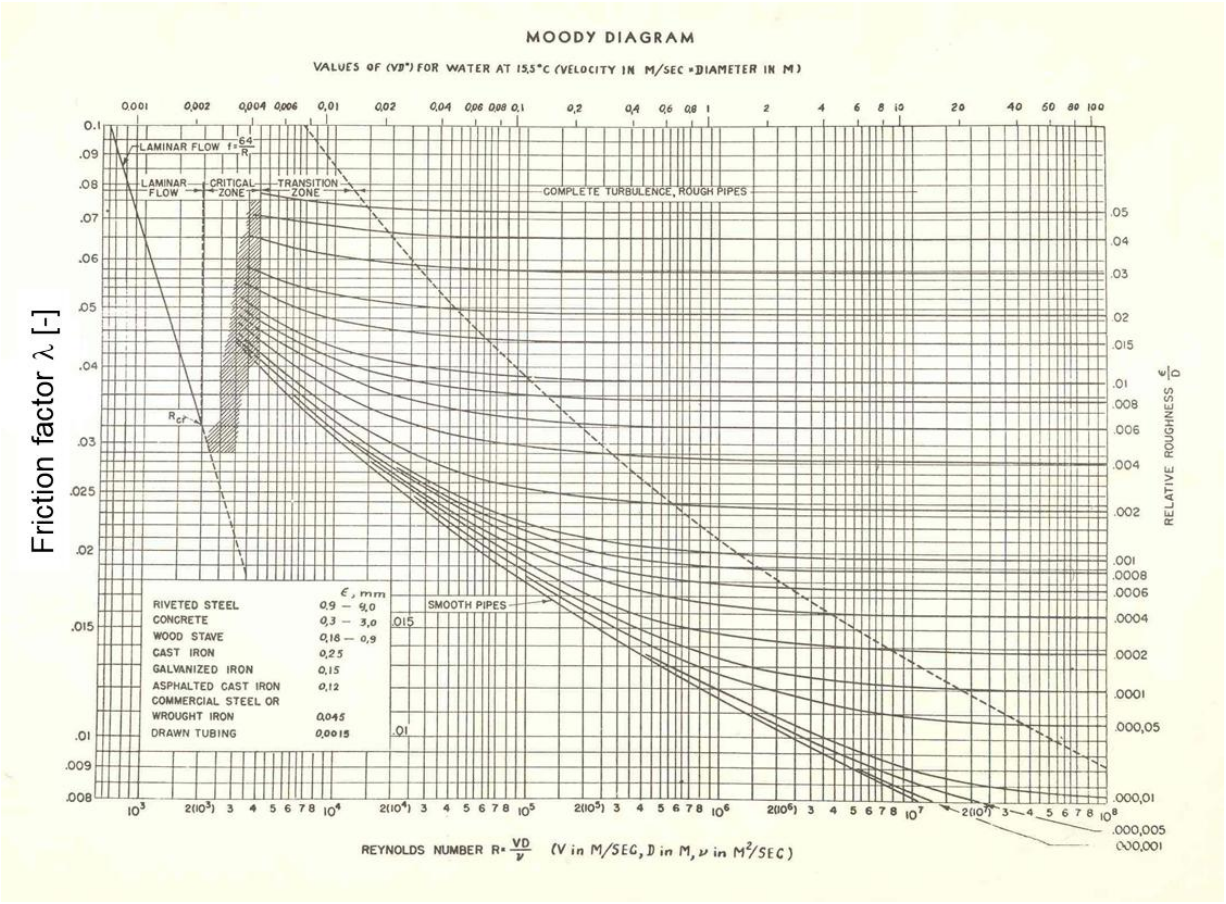
Affinity equations; $\frac{H_1}{n_1^2 \cdot D_1^2} = \frac{H_2}{n_2^2 \cdot D_2^2}$ $\frac{Q_1}{n_1 \cdot D_1^3} = \frac{Q_2}{n_2 \cdot D_2^3}$ $\frac{P_1}{n_1^3 \cdot D_1^5} = \frac{P_2}{n_2^3 \cdot D_2^5}$

Bernoulli's equation on a turbine: $h_{in} + Z_{in} + \frac{C_{in}^2}{2 \cdot g} = H_n + h_{out} + Z_{out} + \frac{C_{out}^2}{2 \cdot g}$

Net Positive Suction Head, NPSH: $NPSH = h_{atm} - h_v + (z_4 - z_2) + (\varsigma_s - 1) \cdot \frac{c_2^2}{2 \cdot g}$

Turbine efficiency: $\eta = \frac{\omega \cdot M}{\rho \cdot g \cdot Q \cdot H_n}$

Rotor Stator Interaction frequency: $RSI = n \cdot Z$


MOODY DIAGRAM
VALUES OF (VD*) FOR WATER AT 15,5°C (VELOCITY IN M/SEC ×DIAMETER IN M)
Friction factor λ [-]
LAMINAR FLOW f=64/R
LAMINAR FLOW
CRITICAL ZONE
TRANSITION ZONE
COMPLETE TURBULENCE, ROUGH PIPES
SMOOTH PIPES
RELATIVE ROUGHNESS ε/D
ε, mm
RIVETED STEEL 0,9 — 9,0
CONCRETE 0,3 — 3,0
WOOD STAVE 0,18 — 0,9
CAST IRON 0,25
GALVANIZED IRON 0,15
ASPHALTED CAST IRON 0,12
COMMERCIAL STEEL OR WROUGHT IRON 0,045
DRAWN TUBING 0,0015
REYNOLDS NUMBER R=VD/ν (V in M/SEC, D in M, ν in M²/SEC)