# Solution Exam 2025

## Problem 1. Turbine and System dynamics

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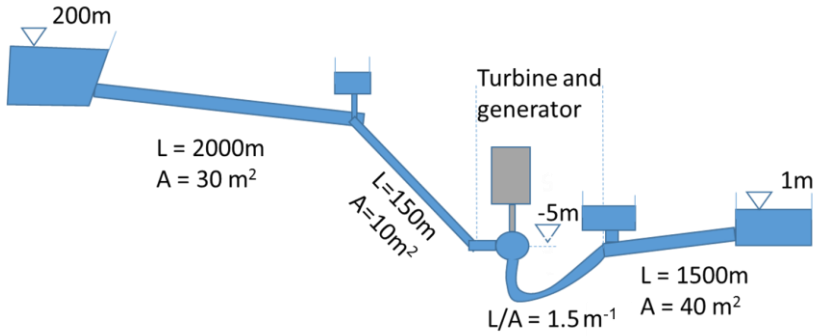
|---|---|---|---|---|
| Input data: | | | | |
| | Head | H | m | 199 |
| | Flow rate | Q | $m^3/s$ | 30 |
| | Number of pole pairs | Poles | | 7 |
| | Upsurge | $\Delta z$ | m | 10 |
| | Gravity contstant | g | $m/s^2$ | 9,81 |
| | Length of the upper tunnel | L | m | 2 000 |
| | Length of the lower tunnel | L | m | 1 500 |
| | Length of the penstock | L | m | 150 |
| | Area of the upper tunnel | A | $m^2$ | 30 |
| | Area of the lower tunnel | A | $m^2$ | 40 |
| | Area of the penstock | A | $m^2$ | 10 |
| | Area of the surge shaft at the outlet from the draft tube | A | $m^2$ | 150 |
| | L/A for the draft tube | L/A | $m^{-1}$ | 1,5 |
| | Turbine closing time for 1B and 1 C | $T_C$ | s | 11 |
| | Turbine closing time for 1F | $T_C$ | s | 8 |
| | | | | |
| Calculated data: | | | | |
| 1A | Area of the upper surge shaft | $A_{Shaft}$ | $m^2$ | 61 |
| 1B | Pressure in the draft tube | $\Delta H$ | m | 10,8 |
| 1D | Down surge in the surge shaft at the outlet from the draft tube | $\Delta z$ | m | 4,8 |
| | Frequency of U-tube oscillation | $\omega$ | rad/s | 0,042 |
| 1E | Period of U-tube oscillation | T | s | 150 |
| 1F | Waterhammer upstream the turbine | $\Delta H$ | m | 31 |



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|---|---|---|---|
| Velocity in the tunnel | c | m/s | 1 |

## Problem 2. Head Loss

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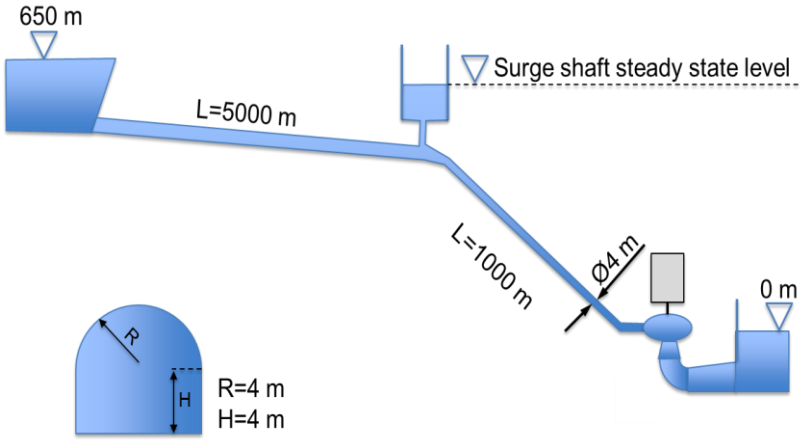
| Input data: | | | | |
| | Gross Head | $H_G$ | m | 650 |
| | Flow rate | Q | $m^3/s$ | 150 |
| | Gravity contstant | g | $m/s^2$ | 9,81 |
| | Length of the upper tunnel | L | m | 5 000 |
| | Radius of the tunnel | R | m | 4 |
| | Height of the tunnel | H | m | 4 |
| | Length of the penstock | L | m | 1 000 |
| | Diameter of the penstock | D | m | 4,0 |
| | Turbine efficiency | $\eta_T$ | | 0,94 |
| | Generator efficiency | $\eta_G$ | | 0,98 |
| | Transformer Efficiency | $\eta_{Tr}$ | | 0,98 |
| | Mannings number | M | $m^{1/3}/s$ | 34 |
| | Kinetic viscosity | $\nu$ | $m^2/s$ | 1,308E-06 |
| | Roughness of the penstock | $\varepsilon$ | m | 0,0004 |

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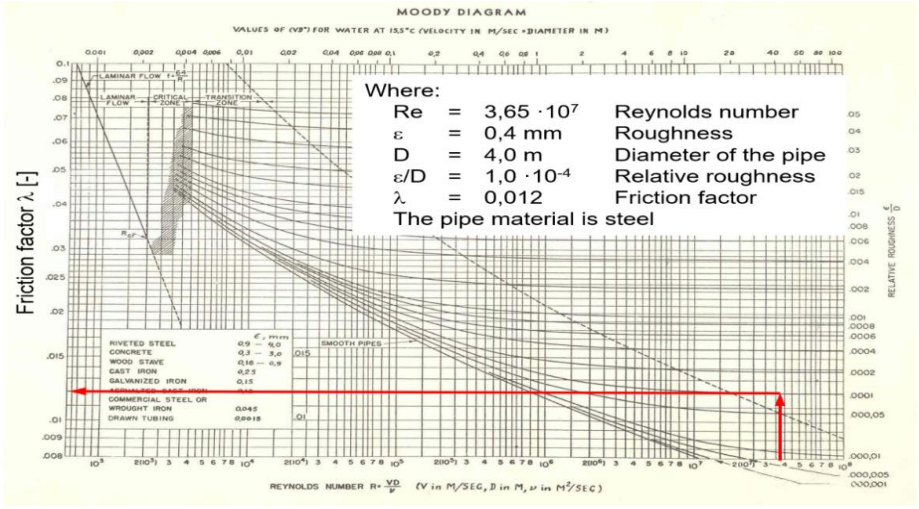
|---|---|---|---|---|
| Calculated data: | | | | |
| | Turbine power output | $P_T$ | MW | 853 |
| | Generator power output | $P_G$ | MW | 836 |
| | Transformer power output | $P_{Tr}$ | MW | 819 |
| | Area of the tunnel | A | $m^2$ | 57,1 |
| | Area of the penstock | A | $m^2$ | 12,6 |
| | Velocity in the tunnel | C | m/s | 2,63 |

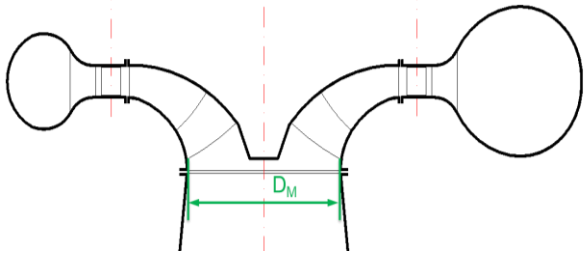
| | | | |
|---|---|---|---|
| Velocity in the penstock | C | m/s | 11,9 |
| Perimeter of the tunnel | P | m | 28,6 |
| Hydraulic radius | $R_h$ | m | 2,00 |
| Head loss in the tunnel | $H_f$ | m | 11,83 |
| Reynolds number in the penstock | Re | | 3,65E+07 |
| Relative rougness of the penstock | ε/D | | 1,00E-04 |
| Friction factor fromm Moody's diagram | λ | | 0,012 |
| Head loss in the penstock | $H_f$ | m | 21,8 |
| Level in the surge shaft | | m | 638,2 |
| Net head | $H_n$ | m | 616,4 |

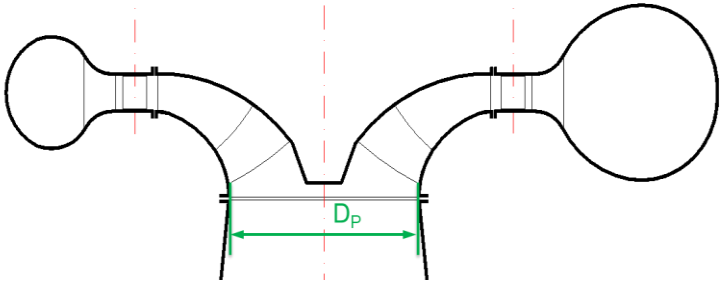


# Problem 4. Francis turbine performance

| | | | | |
|---|---|---|---|---|
| Input data: | | | | |
| | Inlet pressure on the model turbine | $p_{inlet}$ | kPa | 235 |
| | Outlet pressure on the model turbine | $p_{outlet}$ | kPa | 95 |
| | Atmospheric pressure | $p_{atm}$ | kPa | 93 |
| | Flow rate on the model turbine | $Q_M$ | $m^3/s$ | 0,245 |
| | Gravity contstant | g | $m/s^2$ | 9,80 |
| | Density of water | $\rho$ | $kg/m^3$ | 1 000 |
| | Velocity at the inlet of the turbine | $C_{inlet}$ | m/s | 3 |
| | Velocity at the outlet of the turbine | $C_{outlet}$ | m/s | 1 |
| | Level at the inlet | $Z_{inlet}$ | m | 1,5 |
| | Level at the outlet | $Z_{outlet}$ | m | 1 |
| | Model Turbine Torque | $M_T$ | Nm | 900 |
| | Model Turbine Speed | $n_M$ | rpm | 350 |
| | Prototype Turbine Speed | $n_P$ | rpm | 375 |
| | Generator efficiency | $\eta_G$ | | 0,99 |
| | Model Turbine runner outlet diameter | $D_M$ | m | 0,35 |
| | Prototype Turbine runner outlet diameter | $D_P$ | m | 1,78 |
| | Number of runner blades | $Z_{runner}$ | | 30 |
| | Number of guide vanes | $Z_{guide\ vanes}$ | | 24 |

Calculated data:

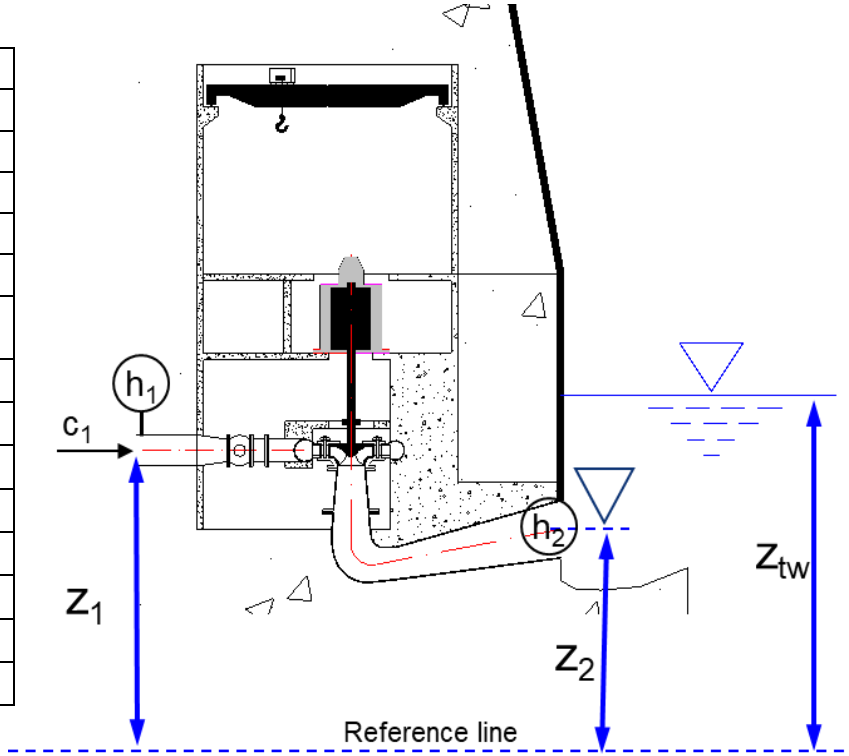


| | | | |
|---|---|---|---|
| Net Head on the model turbine | $H_n$ | m | 15,2 |
| Turbine Speed | $\omega$ | rad/s | 36,7 |
| Turbine Speed - reduced | $\underline{\omega}$ | | 2,1 |
| Flow rate on the model turbine - reduced | $\underline{Q_M}$ | | 0,014 |
| Speed number | $\Omega$ | | 0,25 |
| Turbine power output | $P_T$ | kW | 33 |
| Turbine efficiency | $\eta_T$ | | 90,4 % |
| Speed factor | $n_{ED}$ | | 0,17 |
| Flow factor | $Q_{ED}$ | | 0,16 |
| Flow rate on the Prototype turbine | $Q_P$ | $m^3/s$ | 35 |
| Net Head on the prototype turbine | $H_n$ | m | 454 |
| Rotor Stator Interaction measured in the model runner | RSI | Hz | 140 |

## Problem 5. Francis turbine net Head

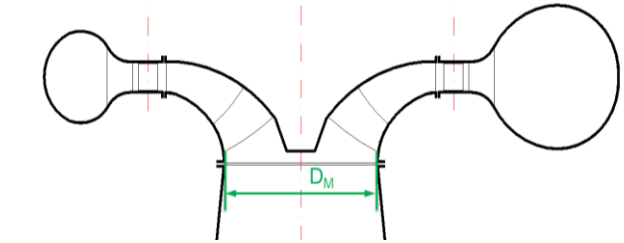
| | | | | |
|---|---|---|---|---|
| Input data: | | | | |
| | Flow rate | Q | $m^3/s$ | 100 |
| | Gravity contstant | g | $m/s^2$ | 9,81 |
| | Density of water | ρ | $kg/m^3$ | 1 000 |
| | Diameter of the inlet pipe | D | m | 4 |
| | Cross section at the outlet of the draft tube | A | $m^2$ | 50 |
| | Level at the inlet to the turbine | $z_1$ | m | 100 |
| | Level at the outlet from the draft tube | $z_2$ | m | 95 |
| | Tail Water Level | $z_{TWL}$ | m | 115 |
| | Static pressure at the inlet to the turbine | $h_1$ | m | 450 |
| | Atmospheric pressure | $p_{atm}$ | kPa | 100 |
| | Turbine efficiency | $\eta_T$ | | 0,94 |
| | Generator efficiency | $\eta_G$ | | 0,98 |
| | Transformer Efficiency | $\eta_{Tr}$ | | 0,98 |

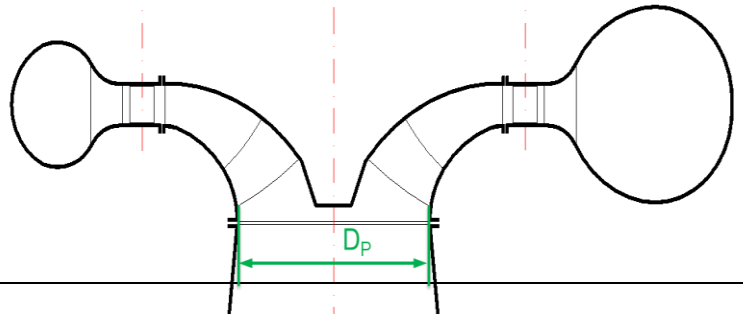



| | | | | |
|---|---|---|---|---|
| Calculated data: | | | | |
| | Velocity at the inlet to the turbine | $C_1$ | m/s | 8,0 |
| | Velocity at the outlet of the turbine | $C_2$ | m/s | 2,0 |
| | Atmospheric pressure | $h_{atm}$ | m | 10,2 |
| | Static pressure at the outlet from the draft tube | $h_2$ | m | 20,0 |
| | Net Head | $H_n$ | m | 438 |
| | Generator power output | $P_G$ | MW | 396 |
| | Submergence of the turbine | $H_s$ | m | 15 |

## Problem 6.

| | | | | |
|---|---|---|---|---|
| Input data: | | | | |
| | Gravity contstant | $g$ | $m/s^2$ | 9,81 |
| | Density of water | $\rho$ | $kg/m^3$ | 1 000 |
| | Net Head on the model turbine | $H_{nM}$ | m | 14,6 |
| | Net Head on the prototype turbine | $H_{nP}$ | m | 377 |
| | Model Turbine Speed | $n_M$ | rpm | 350 |
| | Speed factor at best efficiency point | $n_{ED}$ | | 0,17 |
| | Flow factor at best efficiency point | $Q_{ED}$ | | 0,16 |
| | Model Turbine runner outlet diameter | $D_M$ | m | 0,35 |
| | Prototype Turbine runner outlet diameter | $D_P$ | m | 1,86 |
| | Number of runner blades | $Z_{runner}$ | | 17 |
| | Number of guide vanes | $Z_{guide\ vanes}$ | | 23 |
| | | | | |
| Calculated data: | | | | |
| | Prototype turbine speed | $n_P$ | rpm | 333,50 |
| | Prototype flow rate | $Q_P$ | $m^3/s$ | 33,7 |
| | Flow rate on the prototype turbine - reduced | $\underline{Q_P}$ | | 0,4 |
| | Turbine Speed | $\omega$ | rad/s | 34,9 |
| | Turbine Speed - reduced | $\underline{\omega}$ | | 0,41 |
| | Speed number | $\Omega$ | | 0,25 |
| | Rotor Stator Interaction measured in between the guide vanes and the runner | RSI | Hz | 94 |

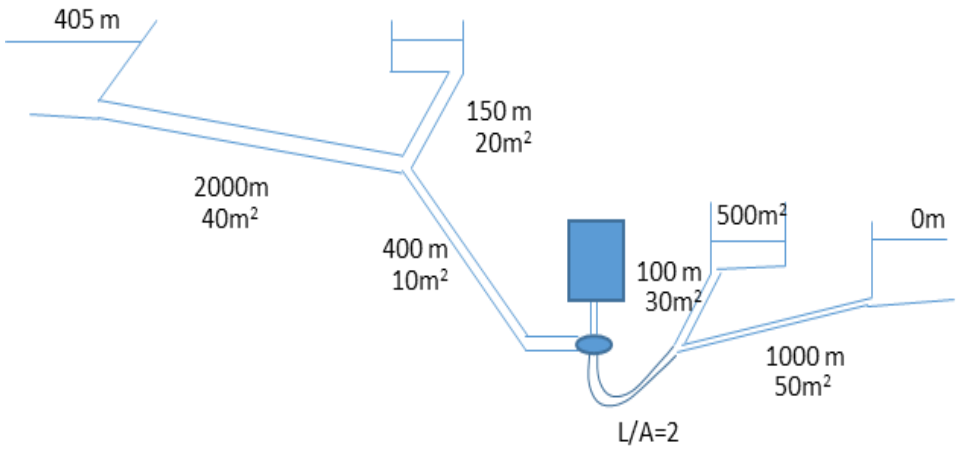


# Problem 7 System Dynamics

Given Data

| | Symbol | Unit | |
|---|---|---|---|
| Gravity contstant | g | $m/s^2$ | 9,81 |
| Head | H | m | 400 |
| Flow rate | Q | $m^3/s$ | 40 |
| Upper reservoir level | $z_{UR}$ | m | 405 |
| Lower reservoir level | $z_{LR}$ | m | 0 |
| Tunnel length | $L_{Tunnel}$ | m | 2000 |
| Tunnel Area | $A_{Tunnel}$ | $m^2$ | 40 |
| Length of penstock | $L_P$ | m | 400 |
| Area of penstock | $A_P$ | $m^2$ | 10 |
| L/A of draft tube | L/A | $m^{-1}$ | 2 |
| Tailrace tunnel length | $L_{tailraice}$ | m | 1000 |
| Tailrace tunnel area | $A_{tailrace}$ | $m^2$ | 50 |
| Upper surge pipe length | $L_{USP}$ | m | 150 |
| Upper surge pipe area | $A_{USP}$ | $m^2$ | 20 |
| Lower surge pipe length | $L_{LSP}$ | m | 100 |
| Lower surge pipe area | $A_{LSP}$ | $m^2$ | 30 |
| Number off pole pairs in the generator | | - | 6 |
| Closing time of wicket gate | $T_C$ | s | 8 |
| Upsurge for question 7D | $\Delta z$ | m | 10 |
| Acceleration time of the generator | $T_a$ | s | 5 |



| | | | | |
|---|---|---|---|---|
| | Area of lower surge shaft | $A_{LSS}$ | $m^2$ | 500 |

Calculated values

| | | | | | |
|---|---|---|---|---|---|
| 1A | Speed of generator | n | rpm | 500 | |
| 1B | Pressure rise at turbine | $\Delta h$ | m | 24,21 | |
| 1D | Area of surge shaft | Ass | $m^2$ | 93,8 | |
| 1E | Penstock time constant | Tw | s | 0,54 | Stable |
| | Frequency of Tail water surge | | Hz | 0,029 | |
| 1F | Time period of Tail water surge | T | s | 216,68 | |