Mindre utdanningsprosjekt ved NV

Rapport og erfaringsdeling

Prosjektnavn: ***Innføring av mer IKT i problemløsning innen Optikk TFY4195***

Prosjektleder(e): Prof Mikael Lindgren

Prosjektperiode: V2021

Tildeling fra NV: 30.000

Hva ble midlene brukt til?

3 studenter fikk til oppgave å utvikle innleveringsoppgaver innen optikk. Sammen med prosjektleder hadde vi møter gjennom vårsemestret der hvert student fikk i oppgave å lage to oppgaver som løses med hjelp av Python software. Til hvert oppgave ble også LF laget med detaljert kode. Gruppen jobbet felles med oppgavene så noen studenter fikk prøve løse oppgavene som di andre laget, osv.

Eksempel på tema (se oppgavene i vedlegg):

Cartesian oval and refraction at curved surfaces

Polarization of EM waves

The Fresnel Rhomb (and circularly polarized light from reflection)

Interferometry and coherence

Fraunhofer diffraction – single to multiple slits

The diffraction grating

Guided waves in thin layer

Hvordan gikk prosjektet? (F.eks. erfaringer med bruk av digitale verktøy eller undervisningsmetoder, erfaringer som andre faglærere kunne lære av, osv.)

Det ble veldig flotte oppgaver. Noen deloppgaver ble brukt under semestret, og noen er spart til neste år V2022 når kursen skal bygges på med nye labøvinger. Her er det tenkt å bruke IKT-oppgavene for at i forhånd simulere noen av eksperimentene (f e ks diffraksjon) eller analysere måledata (f eks fra Michelson interferometer). Veldig godt å ha mange oppgaver å jobbe videre med og stoppe inn i kurset neste vår. Studentene likte veldig godt å jobbe med slike ting (de liker å løse problem med dator) og lærte mye selve, og virket meget fornøyde.

Jeg anbefaler slike utdanningsprosjekter for å utvikle 'IKT-profil' på tradisjonell analog problemløsning.

TFY4195: Optics–Assignment XXX

March 2021

1 Cartesian oval

A Cartesian oval (figure 1) is a lens that form perfect images by refraction with no spherical aberration.

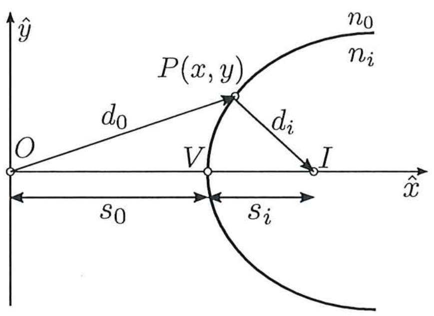

Figure 1: A Cartesian oval.

a) Find the equation for a Cartesian oval where the object lies on the optical x axis 20 cm to the left of the oval ($s_0 = 20$ cm) and the image lies 10 cm inside the the lens ($s_i = 10$ cm). Let the outer medium be air and the lens be made out of glass ($n_1 = 1.5$).

b) Find the radius of curvature for imaging by refraction from a single spherical surface having the same object and image point as in a).

c) Plot the Cartesian oval in a) and the spherical surface in b) together having the same vertex point. What can you say about the accuracy for imaging with the Cartesian oval compared to the spherical surface when considering incoming rays at different angles to the optical axis?

2 Refraction at a spherical surface

In this problem, we will look for an equation for the focal length of a spherical surface without using the paraxial approximation. In figure 2 we can see an incoming ray parallel to the optical axis, refracted at point I a distance h above the optical axis. This point must satisfy the law of refraction

$$n_1 \sin \alpha = n_2 \sin \beta, \tag{1}$$

where α and β are the angles of incidence and refraction, respectively. Let air surround the lens so that $n_2 = 1$.

a) Find, using the trigonometric definitions of $\sin x$ and $\cos x$, an expression for $\sin \gamma$ and $\cos \gamma$ that only depends on $f(h, n, R)$, d and h. You might find these expressions useful in b).

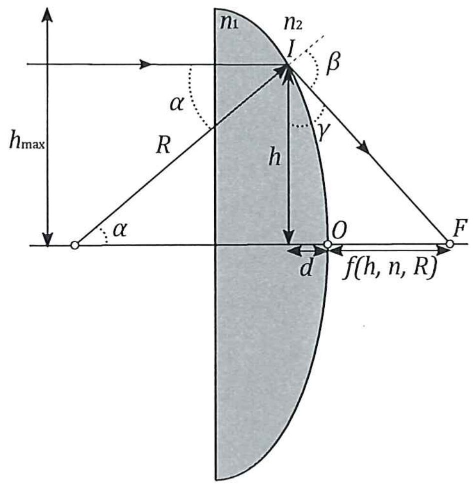

Figure 2: An incoming ray parallel to the optical axis refracted at I by a sphere with radius R and refractive index n_1.

b) Find an expression for $\sin\alpha$ and $\cos\beta$ that only depends on f, d and h and insert those expressions into (1) to get the equation

$$n = \frac{R+f}{\sqrt{(d+f)^2 + h^2}}. \tag{2}$$

Hint: The angular relation $\frac{\pi}{2} - \alpha + \gamma + \beta = \pi$ might come in handy.

c) If one solves (2) for f (you are welcome to try if you are feeling brave), we get that the focal length is equal to

$$f(h,n,R) = R\frac{-n\sqrt{R^2-h^2} + \sqrt{R^2-n^2h^2} + nR}{n\sqrt{R^2-h^2} - \sqrt{R^2-n^2h^2}}. \tag{3}$$

Plot the focal length as a function of the height above the optical axis h of the incoming ray using $R = 2.5$ cm and let the lens be made out of glass ($n_1 = 1.5$). What can you say about the focal length when $h \to 0$?

d) Let us now look at how the focal length f depends on the radius of the lens R. Plot the relationship between f as a function of R at $h = 0$ and $h = 3$. How does this relationship depend on R? Find a numerical value for R if we want the focal length at $h = 3$ cm to be at least 90% of the value at $h = 0$. What does these results say about the choice of lenses used in the lab if we want the focal length to stay as constant as possible for different h?

3 Dispersion

In reality, n is not a constant value but depend on wavelength. This is called dispersion, and you will learn more about this later in the course when you look at EM-waves. Assume normal dispersion, so that

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ..., \tag{4}$$

where $A, B, C, ...$ are known as the Sellmeier coefficients. How will f look with varying n for red ($\lambda = 620$ nm), green ($\lambda = 510$ nm) and blue ($\lambda = 470$ nm) light? Plot f as a function of h (like in problem 2 c)) with $R = 2.5$ cm and $n = n(\lambda)$. Use only the first two terms in (4) with $A = 1.4$ and $B = 2.5 \cdot 10^6$ Å^2.

TFY4195: Optics–Assignment 4

February 2021

1 Polarization of EM-waves

Below the electric field vector for some electromagnetic waves are given. Explain what types of polarization we would expect by considering the amplitude and phase difference of the electromagnetic waves. Analyze the polarization state by plotting the electric field components over one period (for example at z = 0).

a) $\vec{E} = E_0 \cos(kz - \omega t - \frac{\pi}{5})\hat{x} \; + \; E_0 \cos(kz - \omega t + \frac{\pi}{6})\hat{y}$

b) $\vec{E} = E_0 \cos(kz - \omega t - \frac{\pi}{4})\hat{x} \; + \; E_0 \cos(kz - \omega t + \frac{\pi}{4})\hat{y}$

c) $\vec{E} = 1.5E_0 \sin(kz - \omega t)\hat{x} \; + \; 0.5E_0 \cos(kz - \omega t - \frac{\pi}{2})\hat{y}$

d) $\vec{E} = 2E_0 \sin(kz - \omega t)\hat{x} \; + \; E_0 \sin(kz - \omega t - \frac{\pi}{2})\hat{y}$

2 The Fresnel Rhomb

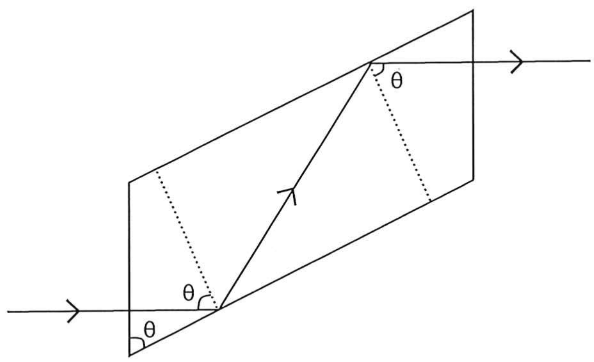

Figure 1: The Fresnel rhomb with apex angle θ.

The Fresnel rhomb (fig. 1) is an optical prism that introduces a 90° phase shift between the TM and TE components of polarized incident light, by the means of two internal reflections inside the prism. In this task, you will use the Fresnel rhomb to produce circularly polarized light from linearly polarized incident light. The refractive index inside the Fresnel rhomb is $n = 1.52$ and $n = 1$ outside. The surfaces where the light enters and leaves is coated with an anti reflective coating so all of the light is transmitted into and out of the prism.

a) If we let linearly polarized light hit the plane of incidence at 90°, what should the apex angle θ be so the TM and TE components of the light becomes circularly polarized after two internal reflections? Solve numerically.

b) What is the phase difference between the TE and TM modes after both internal reflections when the apex angle is 5% above and below the correct value?

Assignment 6: Interferometry and coherence

Introduction

In this problem set you will analyze spectral data collected with a min-spectrometer. Python is recommended but you may use any software for plots and analysis.

It will be handy to create a function that reads .txt-files. If you open a file, you will see that the first column represents the wavelength, while the second column represents number of counts (amplitude). The latter has been normalized to give 1 for the largest amplitude.

Problem 1

Plot the dataset "lysror.txt". This is data collected from a regular fluorescent tube (lysror.txt). At which wavelengths are the peaks located, and what color do they correspond to? Plot the data from the green laser "laserpekere.txt" in the same plot.

Problem 2

Plot the dataset "laserpekere.txt". As we learn from lecture T10A, the coherence length can be written as $\frac{\lambda^2}{\Delta\lambda}$, where λ is the wavelength in the middle of the curve, and $\Delta\lambda$ is the width of the profile of the curve at half of its maximum amplitude FWHM (Full Width Half Maximum)

(a) What is the coherence length of the laser pointer?

(b) Now analyze the lamp spectrum. Cut out the part between 530 and 570 nm also showing green light. From the shape of the curve, estimate the half width and center wavelength, and calculate the approximate coherence length.

(c) In lecture T10A it was shown how the coherence length could be simulated in a Michelson interferometer by superimposing wavelength contributions from a spectral profile staring from equation:

$$I(s) = 2\int_0^{\omega} W(\omega)[1 + \cos(\frac{\omega s}{v})]\, d\omega \tag{1}$$

$W(\omega)$ is the spectrum, but in angular frequency. By substituting the argument in the cosine, $\frac{\omega s}{v} = \frac{2\pi f s}{c} = \frac{2\pi s}{\lambda}$, we can approximate the integral by summing over the spectrum in terms of wavelength points. Note that the integral over the spectrum just gives a constant (its area, that we normalize to 1) and is not so important. The important part of this is in the superposition of the cosines, and we get the following approximation to estimate the coherence length:

$$I(s) = 2[1 + \sum_{\lambda_{start}}^{\lambda_{end}} W(\lambda_i)\cos(\frac{2\pi s}{\lambda_i})] \tag{2}$$

$W(\lambda_i)$ is the amplitude for the particular wavelength λ_i. Write a computer program that superimposes the wavelength contributions following equation (2) using the green part of the lamp spectrum in (b). Plot over a range of s being two times the estimated coherence length. See how it compares with your estimate in (b). Refer to T10A.pdf for inspiration on plotting.

The $I(s)$ is a quantity you measure directly using the Michelson interferometer where you vary the parameter s by shifting the length of one arm. At the lab you will have the possibility to measure the coherence length in terms of the number fringes you observe for a particular light source/spectrum.

Extra problem: Ultrashort laser pulses and their spectra

(This is a voluntary exercise).

Nowadays it is possible to make lasers that can create very short laser pulses. They are important in various spectroscopic and photonic applications. In this problem you will model an ultrashort laser pulse with a Gaussian wavepacket. The laser (Ti:Sapphire) can usually be tuned, but its peak wavelength is at 800nm. At 800 nm the optical period (1 time-cycle) is approximately $\frac{1}{f}$, which we can get from $c = \lambda f \rightarrow \frac{1}{f} = \frac{\lambda}{c} = \frac{800}{3 \cdot 10^8} = 2.67 \cdot 10^{-15}\text{s} = 2{,}67\,\text{fs}$.

A common laser (Coherent MIRA) of this kind has pulses approximately 200 fs long.

(a) Plot the amplitude of the following Gaussian wavepacket with respect to time t: $y = Ae^{-t^2/T^2}e^{-i\omega t}$. Let the amplitude A be 1. T is the pulselength. Compare three lengths of this pulse: 20, 200 and 2000fs.

(b) Calculate the frequency spectrum by taking the Fourier transform of the wavepacket (you may use angular frequency ω or usual frequency f. You may thereafter change variables and plot the same spectrum in wavelength units using $c = \lambda f$. How does the spectrum change with laser pulse length? A very basic property of lasers is that the product of the pulse width in time domain and pulse width in frequency domain (i.e. the spectral width) is constant. In quantum mechanics this is manifested as the Heisenberg uncertainty. In signal processing one defines it as the time-bandwidth-product.

Tips for b): Import fft from scipy. Define sampling frequency. The sampling time interval is the reciprocal of the sampling frequency. The frequency interval is the number of samples divided by the sampling time interval. The two-sided frequency range is created by dividing the number of samples by the frequency interval. Normalize the fast Fourier transform by dividing with the number of samples.

Some more background: https://en.wikipedia.org/wiki/Ti-sapphire_laser
https://en.wikipedia.org/wiki/Femtosecond

Assignment 7: Fraunhofer diffraction of monochromatic light

Introduction

This assignment considers far-field diffraction, where you will simulate and plot the diffraction pattern of various slit arrangements.

Relevant chapter in Pedrotti for this assignment is chapter 11. Consider rectangular slits. The small angle approximation applies here. Set $I_0 = 1$, and let the distance from the slit screen to the observation screen be $L = 5\text{m}$. Choose the scale of the axes appropriately so essential information about the curves are included.

Problem 1: Single slit diffraction

In this exercise, you will simulate several monochromatic light sources in a single slit diffraction setup. Let the wavelength of the waves be 668nm, 613nm, 575nm, 540nm, 505nm, 470nm, 425nm. Each wavelength corresponds to a color of the rainbow, so let the color of each diffraction intensity curve match this.

- Plot the diffraction pattern of a single slit arrangement with slit width $b = 14\mu\text{m}$ with all of the given wavelengths in one single plot.

What do you observe when the wavelength becomes larger?

Problem 2: Double slit diffraction

Let the wavelength of the monochromatic light source be $\lambda = 575\text{nm}$.

Plot the irradiance of a double slit arrangement with

- Slit width $b = 14\mu\text{m}$, slit separation $a = 55\mu\text{m}$
- Slit width $b = 28\mu\text{m}$, slit separation $a = 55\mu\text{m}$

What happens when the slit width b is increased?

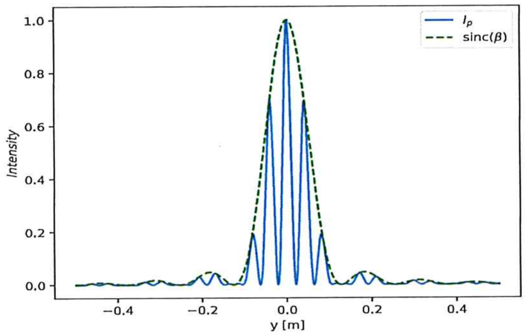

Figure 1: Irradiance of a double slit experiment

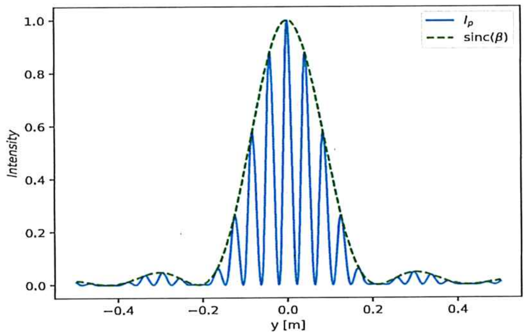

Figure 2: Irradiance of another double slit experiment

Problem 3: Several slit diffraction

Set the slit width $b = 17\mu\text{m}$, and the slit separation $a = 68\mu\text{m}$. λ can still be 575nm. Plot the irradiance pattern of a N-slit arrangement, for $N = 5$ and $N = 8$.

Problem 4

What is the ratio between the slit separation a and the slit width b in Figure 1, and in Figure 2? You can estimate the parameters a and b by analyzing the pattern. Check your result by using the code you made for problem 2.

Voluntary problem: Circular slit

Plot the irradiance of the diffraction pattern of a circular aperture with diameter $D = 0.3\text{mm}$. Let $\lambda = 575\text{nm}$. Where do we expect the first zero intensity circle of the irradiance pattern to occur? Compare this with the diffraction pattern of a single rectangular slit of width $b = 0.3\text{mm}$. Which of the circular slit and rectangular slit has a central maximum with the largest width?

Tips: Python has a scipy module that includes the Bessel function.

Diffraction grating assignment

In this assignment we will be looking at diffraction grating as described in chapter 12 in Pedrotti[3]. The intensity of light for a grating with N grooves is given by

$$I_p(\theta) = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2.$$

With $\beta = \frac{1}{2}kb\sin\theta$ and $\alpha = \frac{1}{2}ka\sin\theta = \frac{a}{b}\beta$, where $k = \frac{2\pi}{\lambda}$ is the wave number, a is the slit gap and b is the slit width. In this assignment we will assume that the range of visible light is exactly 400-700nm.

Part 1

(a) Make a program that plots the intensity with respect to the angle for any given input parameters.

(b) Use parameters $N = 42$, $a = 10^{-5}$m and $b = \frac{a}{7}$. Plot the intensity for $\lambda = 400, 500, 600$ and 700nm in the same plot.

Part 2

Two important properties of diffraction gratings are the resolution and free spectral range. Both of these need to be considered in any experiment involving gratings.

(a) Using the same parameters as in 1b) ,what is the maximum order that has no overlap with any higher or lower orders in the visible light spectrum? Can we change the parameters of the grating to increase/decrease this maximum order, why/why not?

(b) What is the free spectral range for visible light for orders 1, 3 and 5? (Pedrotti[3] 12-2)

(c) What is the minimum resolvable wavelength separation for this grating in the first order with $\lambda = 500$nm? (Pedrotti[3] 12-4)

(What is the minimum value of $\Delta\lambda$ such that the intensity of 500nm and 500nm+$\Delta\lambda$ create two distinct peaks in the first order?)

(d) How many grooves (N) does a grating need if we want $\Delta\lambda$ from (c) to be 0.1nm? Does this seem like a reasonable number of grooves to have?

(e) Use the N from (d), plot the first order peaks for $\lambda = 500$ and 500.1nm along with the sum of their intensities. Do these wavelengths create two distinct peak in the intensity sum?

(f) Repeat (e) for $\lambda = 500$nm and 500.08nm. Are the peaks still distinct?

TFY4195 Optics: Guided Waves

Assignment 5

Part 1

In the video lecture VL8C we discussed and solved for the E_y and H_z-field for a slab waveguide in TE-mode. Make a computer program that solves the mode dispersion equation

$$\frac{2\pi b}{\lambda}\sqrt{n_1^2 - N^2} = 2\arctan\sqrt{\frac{N^2 - n_2^2}{n_1^2 - N^2}} + m\pi$$

numerically for N, for any given b, λ, n_1, n_2 and m. It might be useful to graph both sides of this equation with respect to N.

(a) Use $n_1 = 1.9$, $n_2 = 1.45$, $\lambda = 1.55\,\mu\text{m}$ and a suitable slab thickness, b, that only allows a single mode. Find the solution and plot the E_y-field for this mode.

(b) How does this graph compare with your expectations and what would you expect a graph with for example $m = 3$ to look like?

(c) With the same refractive indices, find a suitable slab thickness that gives 3-5 modes and plot the E_y-field for all of the modes.

(d) How do your plots look compared to what you expected? (Continuity of the interfaces, number of nodes, etc.)

Part 2

Using the video lecture VL8C and notes T8C.pdf as guidance, derive the equations for H_y of the TM-mode. The derivation is very similar, apart from some factors, can you spot any meaningful differences?
Show that the matrix equation now becomes

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & \cos Kb & \sin Kb & -1 \\ n_1^2\gamma & 0 & -n_2^2 K & 0 \\ 0 & -n_2^2 K \sin Kb & n_2^2 K \cos Kb & n_1^2\gamma \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(a) Repeat 1a) but now with the H_y-field for TM-mode.

(b) Find a suitable slab thickness and plot the H_y-field for one m between 3 and 5.

Bonus tasks (voluntary)

Do you notice anything about the continuity of the H_y-field and its derivative? What about the continuity of the E_x- and E_z-fields?